THE DESIGN OF BITUMINOUS MIXTURES WITH CURVED MOHR ENVELOPES

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INTRODUCTION

This paper outlines the results of some further study that has been undertaken since the presentation on this same general topic was made at last year's meeting (1).

First of all, the principles of design for the stability of bituminous mixtures with straight Mohr envelopes will be briefly reviewed. A somewhat different method for the design of bituminous mixtures with curved Mohr envelopes than that outlined a year ago will then be described. This newer approach is being presented partly because it is basically different in several respects, and partly because of certain limitations that have been found for the method described at the last annual meeting.

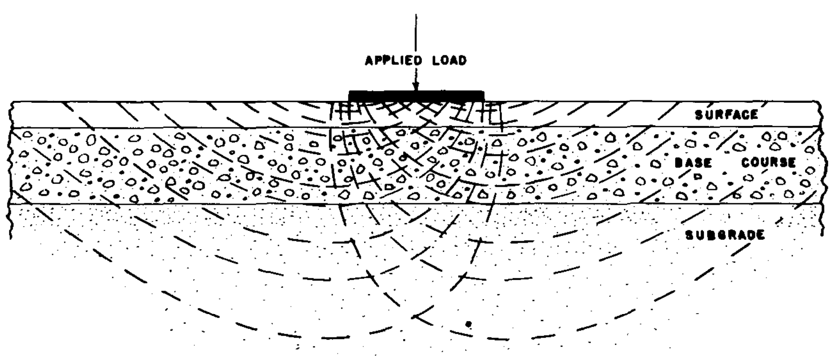

Fig. 1. Diagram of Shear Planes Under a Loaded Area.

The scope of the problem considered in this paper is illustrated by Figure 1. The problem consists of designing bituminous paving mixtures to have sufficient stability to resist being squeezed out between the loaded tire and the base course. The subgrade and base course are assumed to have sufficient strength to support any wheel load that may be applied to the bituminous surface. The further assumption is made that the paving mixtures

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have been properly designed in every other respect, such as density, workability, asphalt content, durability, etc.

The precise nature of the problem of the stability of bituminous pavements is illustrated in Figure 2. The curve representing the distribution of tire pressure across the transverse axis of the contact area of a pneumatic tire resting on a pavement is shown in Figure 2(a) (2,3).* It is quite clear that the stability

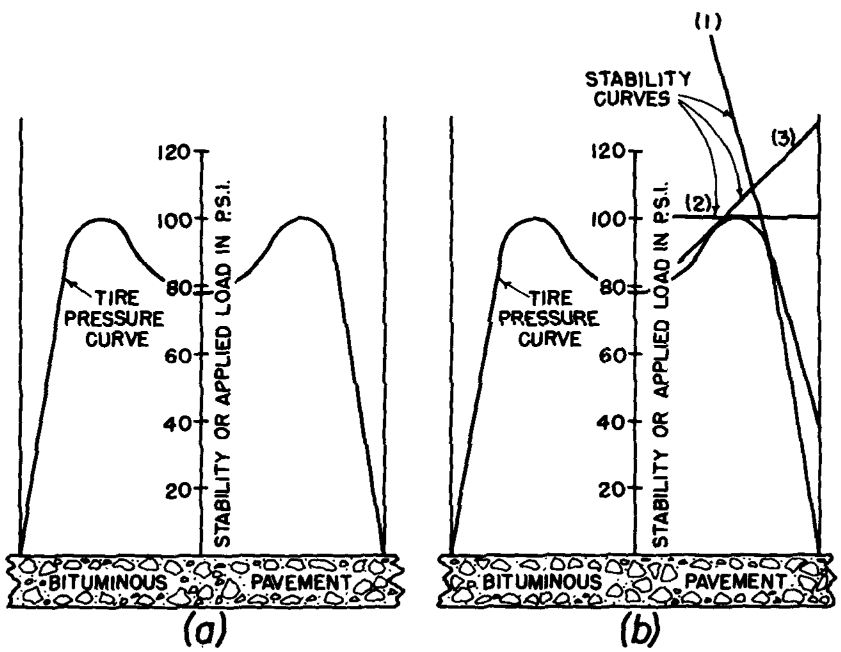


Fig. 2. Diagram Illustrating the Nature of the Stability Problem for Bituminous Paving Mixtures.

developed by the bituminous pavement at all points under the contact area must be at least equal to the tire pressure being applied to each of these points. In Figure 2(b), three possible stability curves just meeting this requirement are shown. It is an axiom of mechanics that a material will fail if the applied load exceeds its ultimate strength. Therefore, if the stability curve for the pavement should cut through the tire pressure curve, Figure 2, there

*See references.

will be a tendency for pavement failure to occur at all points on the contact area for which the stability curve is below the pressure curve. Consequently, the critical stability curve is the one that is just tangent to the tire pressure curve, Figure 2(b).

Three possible critical stability curves of negative (1), zero (2), and positive (3) slope are shown in Figure 2(b). Stability curve (1) indicates that the stability of the pavement is lowest at the edge of the contact area, and increases toward the centre. For stability curve (2), pavement stability is constant from the edge toward the centre of the area loaded by the tire, while for stability curve (3) pavement stability is highest at the edge and decreases toward the centre of the contact area. It will be shown later that for the tire pressure curve illustrated by Figure 2(a), pavement stability appears in general to be represented by stability curve (1); that is, the stability of the pavement increases from the edge toward the centre of the contact area. Furthermore, the stability curve is not a straight line as illustrated in Figure 2(b), but is ordinarily concave upward.

The fundamental problem considered in this paper, and illustrated by Figure 2, is the development of a method of design for any bituminous pavement that will enable the shape and position of its stability curve to be drawn, Figure 2(b), and compared with the tire pressure curve for the severest tire loading anticipated. Safe bituminous pavement design requires that the stability curve be tangent to or above the tire pressure curve for the most severe condition of loading expected.

Before the stability curve for a bituminous pavement can be drawn, the various sources of pavement stability must be determined, and the necessary mathematical equations required to express the stability contribution from each source quantitatively, must be established. When all other factors such as pavement thickness, shape of the curve of tire pressure distribution over the contact area, nature and rate of load, etc. are equal, it is assumed for this paper that the strength of a bituminous pavement depends upon the following three sources of stability:

(a) The inherent stability of the bituminous paving mixture represented by its unconfined compressive strength, $2c\sqrt{\frac{1 + \sin \phi}{1 - \sin \phi}}$.

(b) The increase in pavement stability due to the lateral support L_S provided by the portion of the pavement immediately adjacent to the loaded area

(c) The increase in pavement stability provided by the frictional resistance between pavement and tire and between pavement and base. These two frictional resistances can be represented by an equivalent lateral support L_R, acting in a similar manner to L_S.

A fourth source of pavement stability may exist within some bituminous pavements because of particle interference, direct transfer of load from tire to base through larger particles of aggregate, etc., due to their composition and thickness. This might be referred to as structural stability. For bituminous pavements such as surface treatments and some penetration macadams, that are approximately one aggregate particle in thickness, structural stability could be the major source of pavement stability. For pavements of this type, aggregate particles tend to transfer load directly from tire to base, and there is little or no tendency for the pavement to be squeezed out between the tire and the base course. For the sheet asphalt pavements at the other end of the scale, in which the ratio of pavement thickness to maximum aggregate particle size is usually large, structural stability is probably unimportant and even non-existent, and pavement failure due to lack of stability results in squeezing out between base course and tire. Coarse graded asphaltic concrete mixtures may be intermediate between these two extremes, and structural stability may provide a considerable portion of the stability they develop. On the other hand, for the less coarsely graded asphaltic concretes used for surface courses, and for the stone-filled sheet asphalt, sheet asphalt, and sand asphalt paving mixtures, with which this paper is primarily concerned, and for which the pavement thickness is usually considerably greater than the maximum dimension of the largest particle, it is doubtful that structural stability contributes more than a very minor fraction of the total stability developed by the pavement.

The three major sources of pavement stability just referred to apply equally to bituminous paving mixtures with either straight or curved Mohr envelopes.

RATIONAL DESIGN OF BITUMINOUS MIXTURES WITH STRAIGHT MOHR ENVELOPES

The strength characteristics of bituminous paving mixtures with straight Mohr envelopes are indicated by the magnitudes of the values of cohesion c and angle of internal friction ϕ obtained from the Mohr diagram based upon the triaxial test data for each mixture, Figure 3. One of the objectives of a rational method of design for these paving mixtures consists of determining the smallest corresponding values of c and ϕ needed to provide a

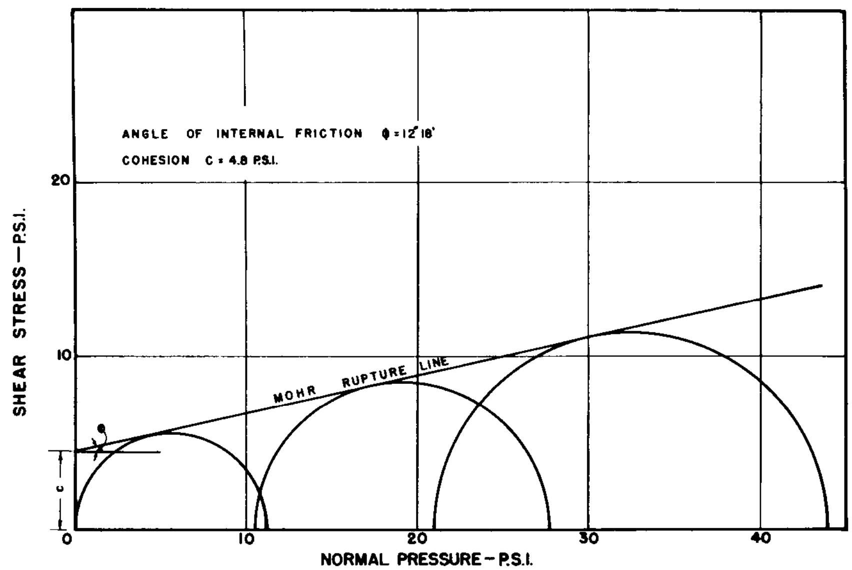


Fig. 3. Typical Mohr Diagram For Triaxial Compression Test.

bituminous pavement that will be stable under the most critical condition of loading (tire pressure curve Figure 2) anticipated throughout its lifetime. The smaller the corresponding values required for c and Ø for the paving mixture, the wider is the range of aggregates from which a selection may be made to provide a bituminous pavement of adequate stability, and this in turn tends to lower the cost of bituminous pavement construction.

General Equation of Stability For a Bituminous Pavement

The stability V (major principal stress) for any Mohr circle on the Mohr diagram, Figure 3, is given by

$$V = 2c\sqrt{\frac{1 + \sin \phi}{1 - \sin \phi}} + L\left(\frac{1 + \sin \phi}{1 - \sin \phi}\right) \qquad (1)$$

Equation (1) represents the general equation of stability for a bituminous pavement in service. The values for c and Ø are provided by the triaxial test. L represents the entire lateral support from all sources applied to the prism of pavement just under the loaded area. Methods for evaluating the two principal sources of lateral support L will be described in the immediately succeeding sections of this paper.

The first term on the right-hand side of equation (1), $2c\sqrt{\frac{1 + \sin \phi}{1 - \sin \phi}}$, represents the unconfined compressive strength of the paving mixture. This is the inherent strength of the bituminous pavement previously referred to, and is one of the three major sources of pavement stability listed earlier in the paper.

Lateral Support of Pavement Adjacent to the Loaded Area

The second term on the right-hand side of equation (1), $L\left(\frac{1 + \sin \phi}{1 - \sin \phi}\right)$, represents the contribution to pavement stability that is made by the two different sources of lateral support L for the prism of pavement just under the loaded area. One of these sources of lateral support is provided by the pavement immediately adjacent to the contact area, and is designated by L_S. Figure 4 indicates that the unconfined compressive strength of the paving mixture can probably be taken as a conservative measure of L_S; that is,

$$L_S = 2c\sqrt{\frac{1 + \sin \phi}{1 - \sin \phi}} \qquad (2)$$

If the value of L_S from equation (2) is substituted for L in equation (1), the following equation results after substitution and simplification:

$$V = \frac{4c}{1 - \sin \phi}\sqrt{\frac{1 + \sin \phi}{1 - \sin \phi}} \qquad (3)$$

A graphical representation of equation (3), in which the c and ϕ requirements for paving mixtures that are to have the indicated values of stability V needed to support a wide range of tire pressures, is provided in Figure 5.

It has been pointed out elsewhere (1,4,5,6), that because Figure 4 is based on strip loading, and therefore neglects several other sources of resistance, the lateral support L provided by the pavement adjacent to the prism of pavement just under the loaded area, is probably greater than the unconfined compressive strength of the paving mixture, and the latter should, therefore, be multiplied by a correction factor K, becoming,

$$L_S = 2c\ K\sqrt{\frac{1 + \sin \phi}{1 - \sin \phi}} \qquad (4)$$

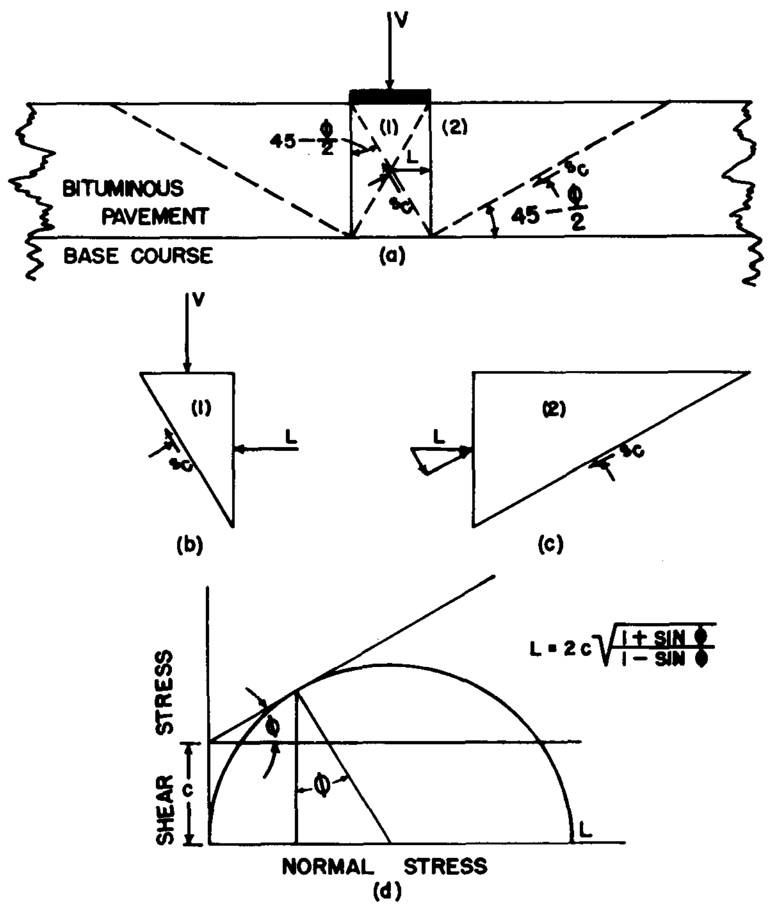


Fig. 4. Illustrating That the Lateral Support L Provided By the Portion of a Bituminous Pavement Surrounding the Loaded Area is Given by $L = 2c\sqrt{\frac{1 + \sin \emptyset}{1 - \sin \emptyset}}$.

which, upon substitution in equation (1) and simplifying, gives

$$V = 2c\sqrt{\frac{1 + \sin \emptyset}{1 - \sin \emptyset}} \left(\frac{K(1 + \sin \emptyset) + 1 - \sin \emptyset}{1 - \sin \emptyset}\right) \qquad (5)$$

Until it can be evaluated more definitely, K = 1 can probably be taken as a conservative value for K, when equation (5) reverts to equation (3).

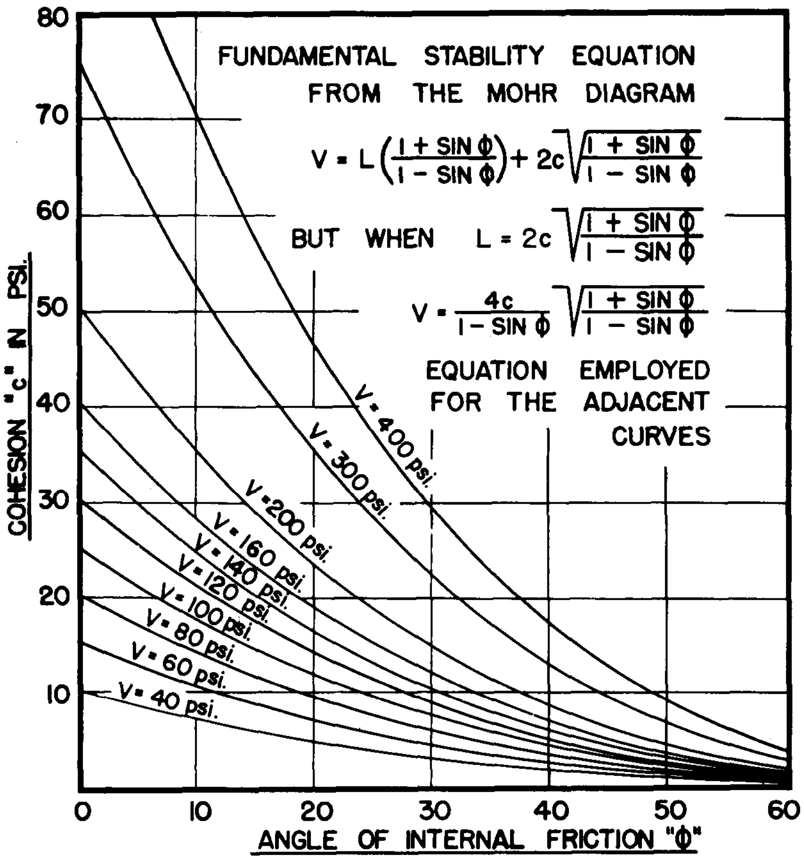


Fig. 5. Design Chart For Bituminous Mixtures Based On Triaxial Test and Values of Lateral Support $L = 2c\sqrt{\frac{1 + \sin\phi}{1 - \sin\phi}}$.

Frictional Resistance between Pavement and Tire and between Pavement and Base

Figure 6 demonstrates that frictional resistance between pavement and tire and between pavement and base seems to provide a second source of lateral support L contributing to pavement stability. Figure 6(a) shows that if a lateral pressure is exerted against a section of pavement between a tire and a base, a shearing stress is developed between pavement and tire and between pavement and base. Conversely, Figure 6(b) illustrates that if sufficient vertical load V is applied by the tire, the pavement tends to be squeezed out between tire and base. In this case, movement of the pavement is resisted by the shearing stresses developed at the interfaces between pavement and tire and between pavement and base. Figure 6(b) also shows that these shearing stresses or frictional resistances between pavement and tire and between pavement and base can be replaced by an equivalent lateral pressure L_R.

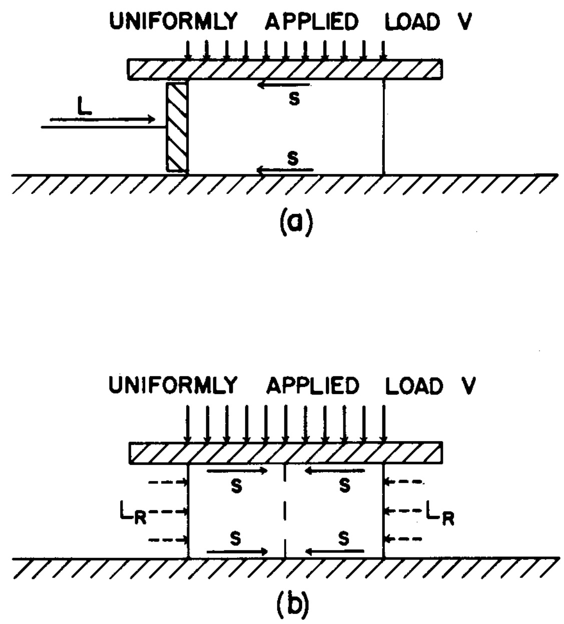
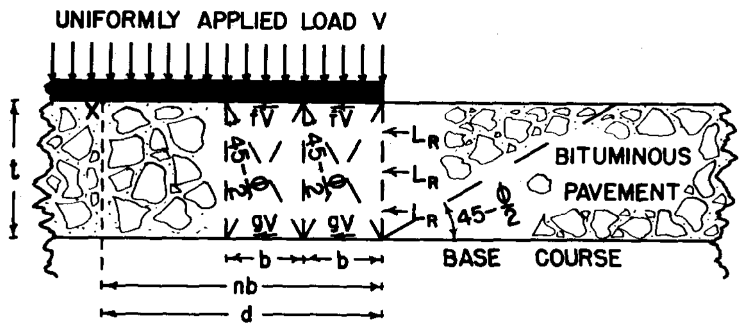


Fig. 6. Diagram Illustrating That Friction between Tire and Pavement and between Pavement and Base is Equivalent To Additional Lateral Support For the Section of Pavement Under a Loaded Area.

Values of the coefficient of friction f between pavement and tire have been measured by Moyer (7) and by Giles and Lee (8). They report values for f up to 1.0 for stationary or slowly moving vehicles, although 0.8 is a more normal top value, with 0.4 to 0.6 as average values. They found that f decreases with increasing vehicle speed. No data appear to be presently available concerning the value of g, the coefficient of friction between pavement and base. Values for f and g must be either determined or assumed for pavement design for each project, if a rational method of design is to be used.

Figure 7 illustrates a method for evaluating the total frictional resistance between pavement and tire and between pavement and base, that is developed at any point X under the loaded area at a



$$fV = P(c + V \tan \phi) \quad \text{WHERE } P \leq 1$$

$$gV = Q(c + V \tan \phi) \quad \text{WHERE } Q \leq 1$$

$$L_R = \frac{d}{t}(P+Q)(c + V \tan \phi)$$

ALSO

$$L_R = \frac{dV}{t}(f+g)$$

Fig. 7. Diagram Illustrating Method For Calculating Value of Lateral Support L_R Equivalent To Frictional Resistance Between Pavement and Tire and Between Pavement and Base.

distance d from the edge of the contact area. Figure 7 shows that these two frictional resistances can be expressed in terms of an equivalent lateral support L_R, where

$$L_R = \frac{d}{t}(P + Q)(c + V \tan \phi) \tag{6}$$

or

$$L_R = \left(\frac{dV}{t}\right) (f + g) \tag{7}$$

Figure 7 demonstrates that the factor P in equation (6) indicates that the maximum frictional resistance fV that can be developed between pavement and tire cannot exceed the shearing resistance of the bituminous pavement itself, which is given by the Coulomb equation $s = c + V \tan \phi$, where V is the pressure applied by the tire to the contact area. The factor Q is of similar significance with respect to the frictional resistance gV between pavement and base. As shown by Figure 7, the highest value that either P or Q can have individually is unity, and the lowest value is zero. Therefore, the maximum value for $P + Q = 2$, and the minimum value is zero.

On the basis of these considerations, the total effective lateral support L provided by a bituminous pavement for the prism of pavement immediately below the contact area can be expressed as

$$L = L_S + L_R \tag{8}$$

from which it follows that equation (1) can be rewritten as

$$V = 2c\sqrt{\frac{1 + \sin \phi}{1 - \sin \phi}} + L_S\left(\frac{1 + \sin \phi}{1 - \sin \phi}\right) + L_R\left(\frac{1 + \sin \phi}{1 - \sin \phi}\right) \tag{9}$$

Since L_S has already been evaluated by equation (4), and L_R by equations (6) or (7), and remembering that the pressure exerted by a tire is not uniform, but varies across the contact area, e.g., Figure 2, equation (9) can be written as follows:

$$V = 2c\sqrt{\frac{1 + \sin \phi}{1 - \sin \phi}} + 2cK\sqrt{\frac{1 + \sin \phi}{1 - \sin \phi}}\left(\frac{1 + \sin \phi}{1 - \sin \phi}\right)$$
$$+ \frac{d}{t}(P + Q)(c + V' \tan \phi)\left(\frac{1 + \sin \phi}{1 - \sin \phi}\right) \tag{10}$$

where

V = stability in p.s.i. developed by the bituminous pavement at any point on the contact area,
c = unit cohesion in p.s.i. obtained from the Mohr diagram,
ϕ = angle of internal friction obtained from the Mohr diagram,

K = a constant which may be taken equal to unity for conservative design,

P = ratio of frictional resistance fV between pavement and tire to the shearing resistance of the pavement represented by the Coulomb equation $s = c + V \tan \phi$, and, therefore, has a maximum value of unity,

Q = ratio of frictional resistance gV between pavement and base to the shearing resistance of the pavement $s = c + v \tan \phi$, and has a maximum value of unity,

d = distance in inches from the edge of the contact area to any point on the contact area where the value of stability V is required,

t = thickness of bituminous pavement in inches, and

V' = the average vertical pressure exerted by the tire between the edge of the contact area and each point on the contact area where the value of stability V is required.

When L_R is represented by equation (7) instead of equation (6), equation (10) becomes

$$V = 2c\sqrt{\frac{1 + \sin \phi}{1 - \sin \phi}} + 2cK\sqrt{\frac{1 + \sin \phi}{1 - \sin \phi}}\left(\frac{1 + \sin \phi}{1 - \sin \phi}\right) + \left(\frac{dV'}{t}\right)(f + g)\left(\frac{1 + \sin \phi}{1 - \sin \phi}\right) \quad (11)$$

in which all symbols have the significance already explained for them.

Evidence that the frictional resistance fV between pavement and tire, and the frictional resistance gV between pavement and base may be important sources of stability for bituminous pavements has been presented in previous papers (1,4,5,6). In addition, field experience and observation have provided many examples of the important influence of good frictional resistance between pavement and base on pavement stability. An otherwise well-designed paving mixture, when laid on a smooth base, or on a base to which it is poorly bonded or not at all, will quickly develop indications of instability under traffic, usually in the form of large tension cracks of well-recognized pattern.

Figure 8 illustrates the application of equations (10) and (11) to the actual design of a bituminous paving mixture. The heavy continuous curve represents the actual pressure applied to the pavement by the tire at all points across the transverse axis of the contact area. The short curves on the right- and left-hand sides of Figure 8 are stability curves for different values of

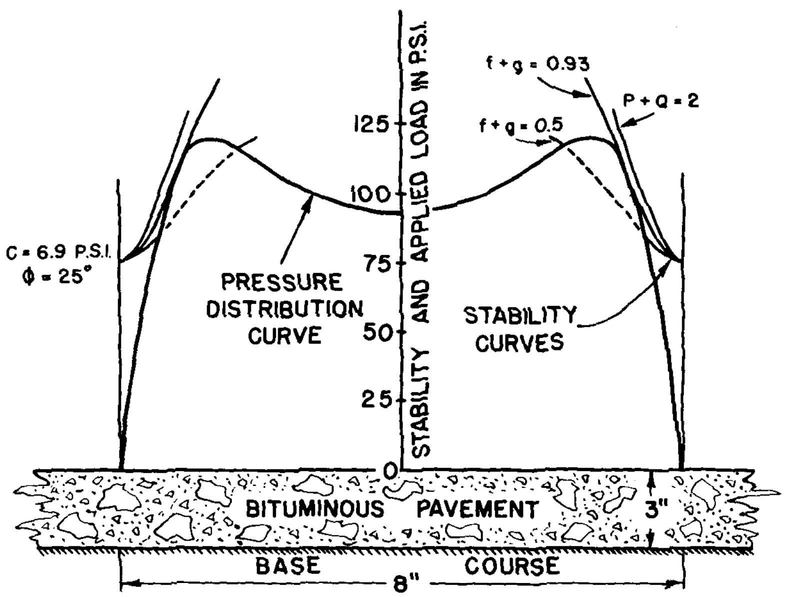


Fig. 8. Influence of Typical Pressure Distribution Over the Contact Area, and of Various Degrees of Frictional Resistance between Pavement and Tire and between Pavement and Base in Terms of f + g Values On the Design of the Underlying Bituminous Pavement (Truck Tire).

f + g for a given paving mixture for which c = 6.9 p.s.i. and $\emptyset = 25^0$, as indicated. The positions of these stability curves are located by applying equations (10) and (11). The location of the stability curve at the edge of the contact area (75 p.s.i.) is calculated by means of the first two terms on the right-hand side of these equations. The increase in stability with increasing distance inward from the edge of the contact area indicated by the stability curves of Figure 8 is due to the frictional resistance between pavement and tire and between pavement and base, and is calculated by means of the third term on the right-hand side of equations (10) and (11), equation (10) being employed for the P + Q = 2 curve, and equation (11) for the f + g = 0.93 and f + g = 0.5 curves. These stability curves indicate that because of these two frictional resistances the pavement can sustain a higher and higher vertical load as the centre of the contact area is approached from the edge; that is, pavement stability increases with increasing distance inward from the edge of the loaded area.

The ordinate axis in the centre of the diagram indicates values for both the tire pressure exerted on the contact area, and the stability developed by the pavement. If it is assumed that the stability of the pavement must be not less than the pressure applied by the tire at any location on the contact area, then it is apparent that the stability curve must not cut through the tire pressure curve at any point. It is equally clear that the critical stability curve is the one that is just tangent to the pressure curve. In Figure 8, the stability curve for f + g = 0.5 cuts through the pressure curve, indicating that the pavement would be unstable for the portion of the contact area for which the stability curve lies below the pressure curve. The stability curve for f + g = 0.93 is just tangent to the pressure curve, and indicates this to be the lowest value of f + g (coefficient of friction between pavement and tire plus coefficient of friction between pavement and base) for which this particular paving mixture (c = 6.95 p.s.i., Ø = 25°) would be stable at all points on the contact area. The stability curve labelled P + Q = 2, on the other hand, indicates the highest f + g values that could be developed by this paving mixture, since any f + g stability curve lying above the P + Q = 2 curve would represent a value of frictional resistance between pavement and tire, or between pavement and base, or both, that was greater than the shearing resistance of the bituminous paving mixture itself. It is apparent that the pavement would fail in shear before such a high value for f, g, or f + g could be developed.

The importance of frictional resistance between pavement and tire and between pavement and base as a source of pavement stability is emphasized in Figure 9, in which stability curves for f + g = 0, f + g = 0.2, f + g = 0.6, and f + g = 1.2 are shown on the right-hand side. These stability curves demonstrate very clearly the decrease in values of c and Ø that is possible as the f + g values are increased. For example, when f + g = 0, a paving mixture with c = 7.8 p.s.i. and Ø = 30°, developing a stability of 107 p.s.i. at the edge of the contact area, is required, while for f + g = 1.2, a bituminous mixture with c = 4.0 p.s.i. and Ø = 30°, developing a stability of only 54 p.s.i. at the edge of the contact area, is adequate; that is, by increasing the f + g value from 0 to 1.2 stability requirements for the paving mixture itself in terms of c and Ø values have been reduced by one-half.

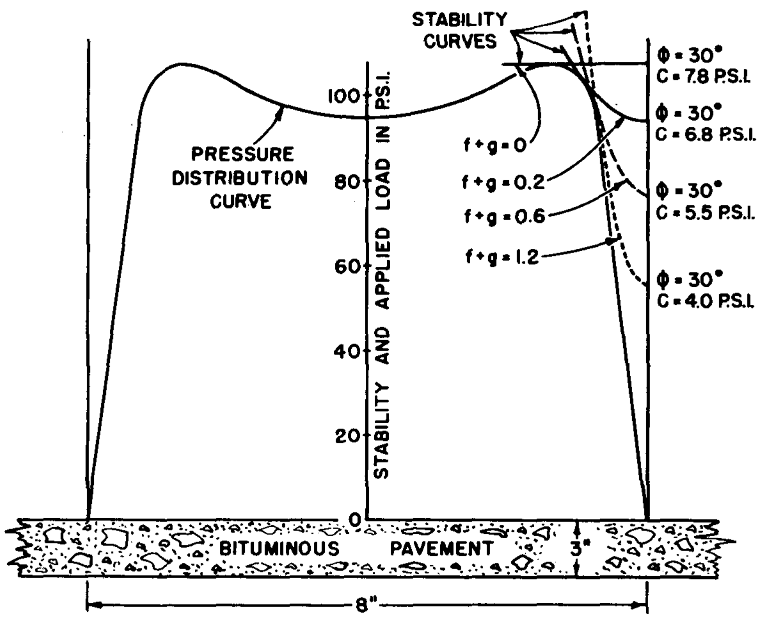


Fig. 9. Illustrating the Influence of Frictional Resistance between Pavement and Tire and between Pavement and Base On the Design of Bituminous Pavements.

Discussion

The influence of other factors such as pavement thickness, shape of the curve of tire pressure distribution over the contact area, nature and rate of loading, etc. on the stability of asphalt pavements in service has already been described elsewhere (1, 4,5,6).

While this portion of the paper provides a theoretical study of the bituminous pavement stability problem as it pertains to paving mixtures with straight Mohr envelopes, the conclusions obtained are in at least qualitative agreement with observations of pavement performance in the field. However, more accurate information is needed concerning the exact shape of the curve of tire pressure distribution across the contact area for the most critical tire loadings on airports and highways, and about the magnitudes of such variables as K, f, g, P, Q, etc., before the overall design equations (10) and (11) can be used with complete confidence, unless fairly large safety factors are to be applied.

Even when such fairly large safety factors are employed, their actual magnitude must remain relatively unknown until each of these variables and its influence on pavement stability can be accurately evaluated.

Nevertheless, if particular care is taken to obtain a strong bond between pavement and base course (a high value for coefficient of friction g), if design is based upon the stationary load condition and for the maximum pressure to be applied to the contact area, and if it is assumed that the factor K = 1, then it is believed that equation (3) or the design curves of Figure 7, which are based on equation (3), provide a conservative basis for bituminous pavement design. That this may be so is verified by the fact that the design curve of Figure 7 for 100 p.s.i. tire pressure requires approximately the same corresponding values for c and Ø as the lower boundary of the satisfactory area of The Asphalt Institute's triaxial design chart, which has been checked with pavement performance in the field (9).

If bituminous pavement design is based upon equation (3) and the design curves of Figure 7, the third term on the right-hand side of equations (9), (10), and (11) is being neglected, and it, therefore, serves as a safety factor. This is well illustrated by

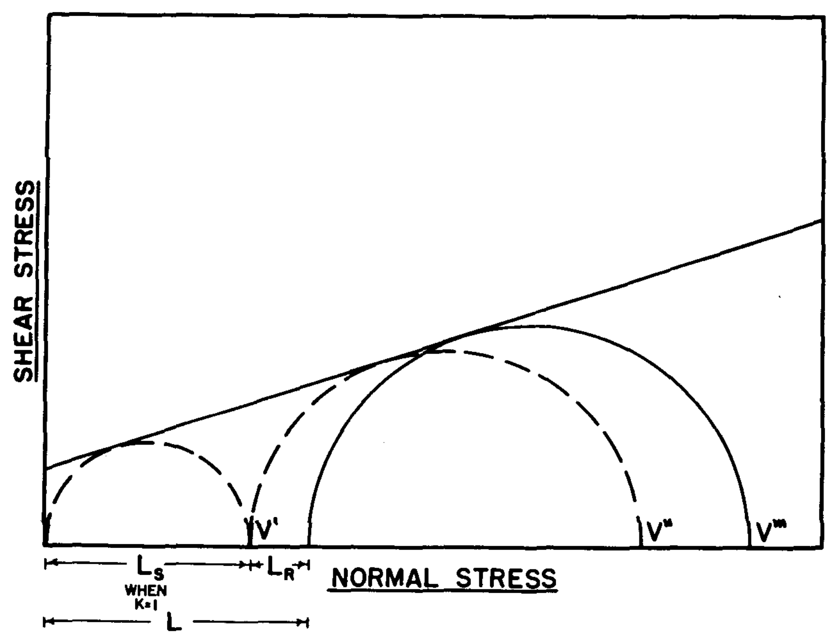


Fig. 10. Influence of L_S and L_R On Pavement Stability.

Figure 10, which by means of a Mohr diagram, represents graphically the general equations of design (1), (3), (9), (10), (11) developed in the first part of this paper. V' is the pavement stability that would result if the only source of pavement stability were the unconfined compressive strength of the paving mixture. V'' represents the pavement stability developed on the assumption that the pavement adjacent to the prism of pavement beneath the loaded area provides the only source of lateral support L, and that $L = L_S$ = the unconfined compressive strength of the paving mixture (K = 1). V''' indicates the pavement stability developed when the lateral support $L = L_S + L_R$. In this case, part of the lateral support L is due to pavement adjacent to the loaded area, L_S, while part is provided by the frictional resistances between pavement and tire and between pavement and base, L_R. Figure 10 demonstrates that neglect of L_R reduces the estimated pavement stability from V''' to V'', and thereby tends to provide the safety factor already referred to, if design is based upon equation (3) and Figure 7.

The design curves of Figure 7 provide bituminous pavement stability requirements for the entire range of tire inflation pressures in common use today, from the low pressure tires of passenger cars to the 300 p.s.i. and higher inflation pressures of the tires on some current jet aircraft.

RATIONAL DESIGN OF BITUMINOUS MIXTURES WITH CURVED MOHR ENVELOPES

The previous section of this paper dealt with the rational design of bituminous pavements constructed with bituminous mixtures with straight Mohr envelopes. The balance of this paper is concerned with the same design problem for bituminous mixtures with curved Mohr envelopes. As was the case last year, no attempt will be made in this paper to explain why the triaxial data for some bituminous mixtures result in curved Mohr envelopes. It is accepted as an experimental fact that most bituminous paving mixtures have Mohr envelopes that appear to be essentially straight lines, but that for some the Mohr envelope is curved. It is further assumed for this paper that the point of contact (tangency) between the curved Mohr envelope and any Mohr circle defines the angle of the plane of failure through the specimen for the particular conditions of stress represented by that Mohr circle. This means that the angle between the plane of failure and the vertical becomes gradually larger (and approaches 45^0 as an asymptote) as the test specimen (or specimens) is (are) subjected to successively greater magnitudes of the principal stresses V and L under incipient failure conditions.

If a rational method of design is to be considered for bituminous pavements constructed from paving mixtures with straight Mohr envelopes, there is equal need for a rational design when the Mohr envelope for the paving mixture is curved. The latter represents the principal objective of the present paper.

When considering the design of bituminous pavements with paving mixtures having curved Mohr envelopes in the paper presented at last year's meeting, it was pointed out that a curved Mohr envelope might be represented by either a parabolic equation (power function), or an exponential equation. Since it seemed to be somewhat easier to use, the development of a rational method of design for bituminous paving mixtures with curved Mohr envelopes contained in last year's paper was based upon representing the curved Mohr envelope by a parabolic or power type of equation. During the discussion of this paper at the meeting a year ago, Dr. Charles Mack suggested a power function equation of somewhat different form for the representation of a curved Mohr envelope.

Comparison of Present Method with Last Year's Approach

Further investigation since last year's paper was published has indicated certain limitations to the power function (parabolic) type of equation for representing a curved Mohr envelope. This is illustrated in Figure 11, in which an attempt has been made to represent the Mohr envelope through the three points X, Y, and Z obtained from triaxial data, by three curves. One of these curves is given by the parabolic equation contained in our last year's paper, another is provided by Dr. Mack's equation of the same general type (a power function), while the third curve results from the use of an exponential equation.

Figure 11 demonstrates that the curves resulting from our last year's parabolic equation, and Dr. Mack's equation, both cut the abscissa to the right of the origin, while the curve given by the exponential equation cuts the abscissa to the left of the origin.

It will be noted from Figure 11 that one extremity of the diameter of the Mohr circle representing the unconfined compressive strength of a bituminous paving mixture must be located at the origin of the Mohr diagram. Figure 11 indicates that when attempting to represent the curved Mohr envelope by either the parabolic or Dr. Mack's equation, curves are obtained that

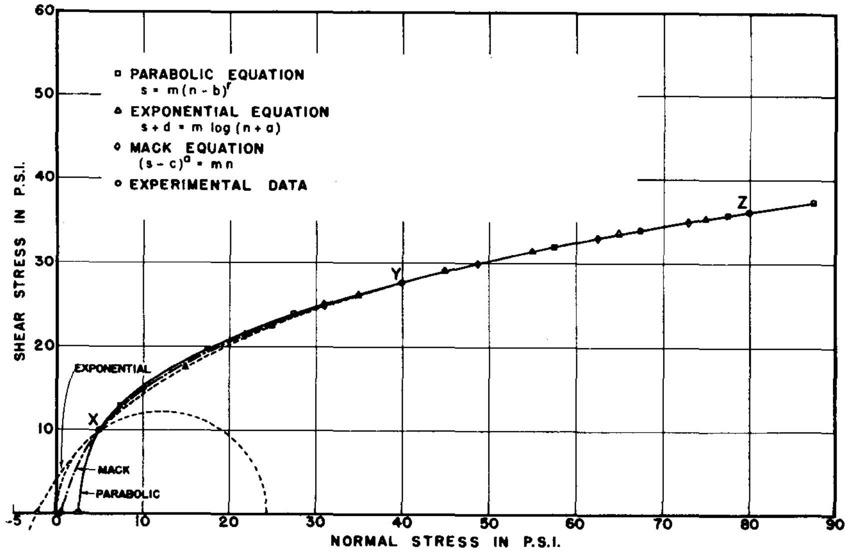


Fig. 11. Mohr Envelopes Provided By Three Different Equations Compared On the Basis of Given Triaxial Data.

intersect the Mohr circle representing the unconfined compressive strength. The envelope to a group of Mohr circles should be tangent to each of them, and no longer satisfies the requirements for an envelope if it cuts through any one of the circles of the group in the manner illustrated in Figure 11. Consequently, for the conditions represented by Figure 11, neither the parabolic equation contained in last year's paper, nor Dr. Mack's equation, is capable of representing the curved Mohr envelope, since the curves for both equations intersect the Mohr circle representing the unconfined compressive strength. On the other hand, the curve provided by the exponential equation is tangent to the Mohr circle representing the unconfined compressive strength, and appears to meet all the other requirements of a curved Mohr envelope.

While our paper for last year's meeting indicated that the curved Mohr envelopes provided by the triaxial data for some bituminous paving mixtures can be satisfactorily represented by a parabolic equation, Figure 11 has shown that this is not always the case. It would appear, therefore, that an exponential equation may be more generally useful for representing a curved Mohr envelope.

The method described in this paper differs from that outlined last year in another important respect. In the paper presented a year ago, a method was proposed that enabled equations (10) and (11), which were derived for the rational design of paving mixtures with straight Mohr envelopes, to be used also for the rational design of paving mixtures with curved Mohr envelopes. This required that the proper values for c and Ø be obtained from Mohr diagrams for bituminous mixtures with curved Mohr envelopes, to substitute in equations (10) and (11). Since c and Ø are not constant for paving mixtures with curved Mohr envelopes, it was necessary to develop a method to provide the particular c and Ø values required. For this purpose, the Mohr circle was determined that represented the stability of the pavement at the edge of the contact area of a loaded tire on the pavement. At the point of contact between this Mohr circle and the Mohr envelope, a tangent was either drawn on the basis of visual judgment, or its precise location and slope were calculated. The c and Ø values given by this tangent were substituted in equations (10) or (11) to determine the stability of the pavement at the edge of the contact area. These c and Ø values were also employed to calculate the position of the stability curve, e.g., Figure 15, on the basis of equations (10) and (11). As indicated in last year's paper, this is an approximate rather than a rigorously accurate method for establishing the stability curve for paving mixtures with curved Mohr envelopes.

The present paper will show that it is not necessary to determine c and Ø values, nor to employ equations (10) and (11) to determine the stability curve for paving mixtures with curved Mohr envelopes. The stability values required can be precisely determined from the mathematical relationships that exist between the curved Mohr envelope and the Mohr circles. No determination of or reference to c and Ø values is made, and equations (10) and (11) are not used.

Design Requirements For Paving Mixtures with Curved Mohr Envelopes

The objective of a rational method of design for bituminous paving mixtures with curved Mohr envelopes (as it is for those with straight Mohr envelopes) is to determine the location of the stability curve for the pavement, and to compare it with the tire pressure curve, e.g. Figures 2, 8, 9, and 15. The stability curve in each of these figures begins at the edge of the contact area and slopes upward toward its centre. Two items of information are, therefore, required:

(1) The stability of the pavement at the edge of the contact area; that is, the location of the beginning of the stability curve.
(2) The stability of the pavement at each point between the edge and the centre of the contact area; that is, the location of the balance of the stability curve.

For bituminous paving mixtures with straight Mohr envelopes, the stability of the pavement at the edge of the contact area is given by the first two terms of the right-hand side of equations (9), (10), and (11), while the stability of the pavement at all points between the edge and the centre of the contact area is determined by means of the third term on the right-hand side of these equations. While it makes no use of these equations, nor of c and Ø values, a roughly similar approach is employed for determining the location of the stability curves for paving mixtures with curved Mohr envelopes.

The procedure for determining, first, the stability of the pavement at the edge of the contact area, and second, the stability of the pavement at all points between the edge and the centre of the contact area, will be briefly described. The development of the equations required, and a sample calculation, are provided in the Appendix.

Pavement Stability At the Edge of the Contact Area

The determination of the stability of a bituminous pavement with a curved Mohr envelope at the edge of the contact area (the position of the stability curve at the edge of the contact area) involves the following steps.

Step No. 1

Corresponding V and L values from the triaxial data are plotted in the form of a principal stress diagram, Figure 12. A smooth curve is drawn through the points either arbitrarily or by methods available for this purpose (10).

Step No. 2.

Several V and L values from the smooth curve of Figure 12 provide Mohr circles for the corresponding Mohr diagrams of Figures 13 and 14. Since Figures 13 and 14 must be kept as simple as possible to illustrate the method of design for bituminous mixtures with curved Mohr envelopes, only two of these Mohr circles are retained in these figures. The values of V and L

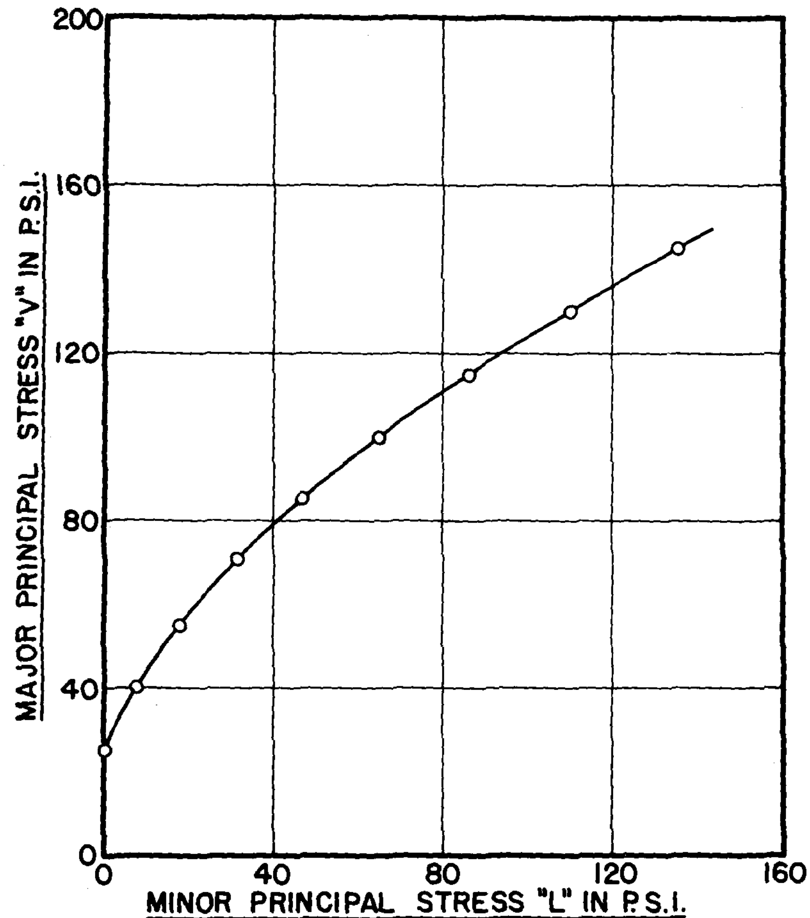


Fig. 12. Principal Stress Diagram For a Bituminous Paving Mixture with A Curved Mohr Envelope.

selected should range from the unconfined compressive strength, L = 0, to somewhat greater than the stability value V likely to be required, which should be large enough to enable the stability curve to be drawn over the required range, e.g. Figure 15(a). Draw a smooth curved line envelope tangent to the several Mohr circles, Figures 13 and 14.

Step No. 3

For this paper it is assumed that the Mohr envelopes of Figures 13 and 14 can be represented by a relatively simple mathematical equation of the exponential type.

$$s + d = m \log (n + a) \tag{12}$$

where

s and n are shear and normal stress, respectively, on the plane of failure, and

d, m and a are constants whose values vary with the location and curvature of the curved Mohr envelopes for different bituminous mixtures.

Equation (12) contains three unknown constants, d, m, and a. These constants can be evaluated by substituting the s and n values for three well-distributed points on the curved line envelope, e.g. X, Y, and Z in Figure 13, into equation (12) and solving

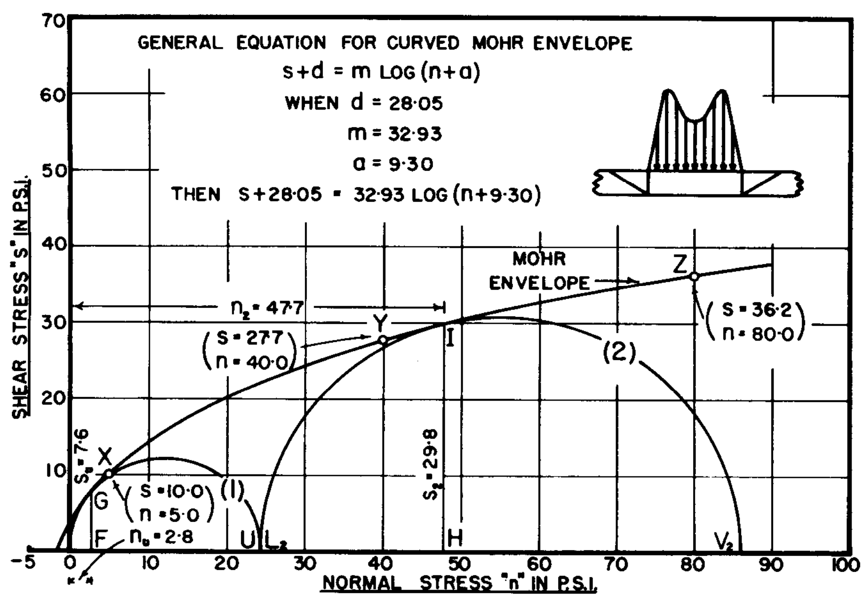


Fig. 13. Illustrating Maximum Vertical Load V Supported By a Bituminous Paving Mixture with a Curved Mohr Envelope When the Lateral Support L is Equal to the Unconfined Compressive Strength of the Material.

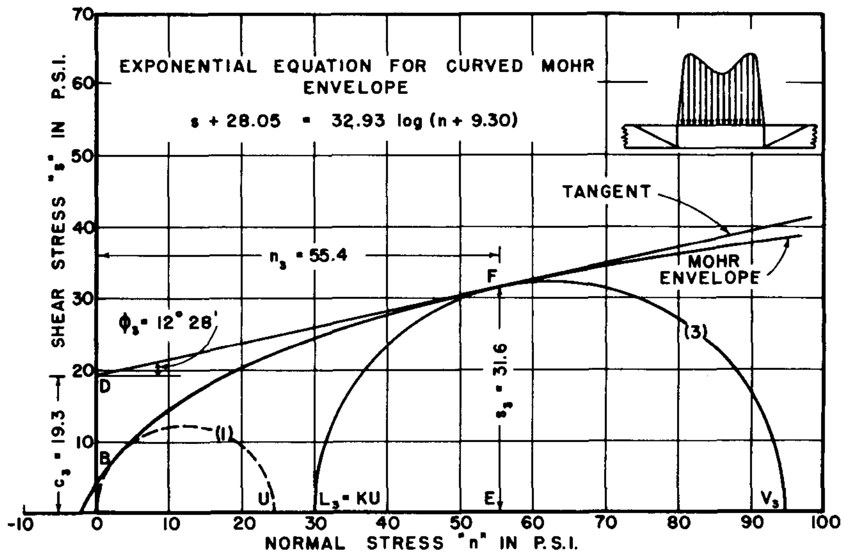


Fig. 14. Illustrating Maximum Vertical Load V That Can Be Carried By a Bituminous Paving Mixture with a Curved Mohr Envelope When the Lateral Support KU Is Either a Fraction or a Multiple of the Unconfined Compressive Strength of the Material (General Case).

the resulting three equations simultaneously (see Appendix). The three points X, Y, and Z should be well distributed over the range from just slightly to the right of the point of tangency between the envelope and the Mohr circle representing the unconfined compressive strength, to somewhat larger than the stability value V required (see Step No. 2 above). The s and n coordinate values for X, Y, and Z should be read as precisely as possible from the smooth curved envelope.

For the curved Mohr envelopes of Figures 13 and 14, the sample calculation worked out in the Appendix gives the following values for d, m, and a:

$$d = 28.05$$
$$m = 32.93$$
$$a = 9.30$$

Figures 13 and 14 illustrate the shape of the curved Mohr envelope given by the exponential equation.

Step No. 4

For the unconfined compressive strength U of the paving mixture, represented by Mohr circle (1) in Figures 13 and 14, the lateral support L is of course zero. The Appendix indicates the following equation for evaluating U:

$$U = \frac{[m \log (n_u + a) - d]^2}{n_u} + n_u \tag{13}$$

and the sample calculation in the Appendix indicates that for Figures 13 and 14

$$U = 24.49 \text{ p.s.i.}$$

Step No. 5

When the lateral support, $L_S = L_2$, provided by the pavement adjacent to the loaded area, is equal to the unconfined compressive strength of the paving mixture U, Mohr circle (2) in Figure 13, the Appendix gives the following equation for the stability of the pavement, V_2, at the edge of the loaded area.

$$V_2 = \frac{\{m \log (n_2 + a) - d\}^2}{n_2 + u} + n_2 \tag{14}$$

When this lateral support $L_S = L_3$ is equal to KU, the unconfined compressive strength U multiplied by a factor K, Mohr circle (3) in Figure 14, the following equation for the stability of the pavement at the edge of the loaded area is given in the Appendix:

$$V_3 = \frac{\{m \log (n_3 + a) - d\}^2}{n_3 + u} + n_3 \tag{15}$$

Consequently, for the paving mixture represented by the triaxial data of Figure 12 and the Mohr diagram of Figure 13, and for the specific case where the lateral support L_S provides by the pavement adjacent to the loaded area is equal to the unconfined compressive strength U, the stability of the pavement at the edge of the loaded area, as given by the sample calculation in the Appendix, is $V_2 = 85.9$ p.s.i.

For the more general case, Figure 14, where the lateral support L_S provided by the pavement adjacent to the loaded area is equal to KU, the unconfined compressive strength multiplied by the factor K, where K may be greater than, equal to, or less than unity, the stability of the pavement at the edge of the loaded area, as given by the sample calculation in the Appendix for the particular conditions of Figure 14, is $V_3 = 94.68$.

Depending upon the conditions they represent in each case, either V_2 or V_3 provide the position of the stability curve at the edge of the contact area.

Pavement Stability between the Edge and Centre of the Contact Area

The increase in pavement stability between the edge and centre of the contact area is due to frictional stresses between pavement and tire and between pavement and base, which oppose the tendency of the applied load to squeeze the pavement out between the tire and the base course. These frictional resistances are equal to an equivalent lateral support L_R, which is expressed by equation (7),

$$L_R = \frac{d\,V'}{t}(f + g) \tag{7}$$

where V' is the average tire pressure acting on the contact area over a distance d from the edge of the contact area, and the other symbols have the significance previously ascribed to them. The stability of the pavement at any point between the edge and the centre of the contact area can be determined by the following steps.

Step No. 6

By means of equation (7), evaluate L_R for an element of pavement at a small definite distance inward from the edge of the contact area, e.g. 1/2 inch from the edge.

Step No. 7

Add the value of L_R obtained by Step No. 6 to the value of L_S employed in Step No. 5 to calculate the stability V_2 or V_3 of the pavement at the edge of the contact area.

Step No. 8

Since $L_S + L_R = KU$, where U is the unconfined compressive strength of the paving mixture, the value of stability V can be calculated from equation (15); that is, the value of stability V at one-half inch inward from the edge of the contact area is given by

$$V = \frac{\{m \log (n + a) - d\}^2}{n - KU} + n \qquad (15)$$

Step No. 9

The stability V at other distances d inward from the edge of the contact area can be similarly calculated by Steps Nos. 6, 7,

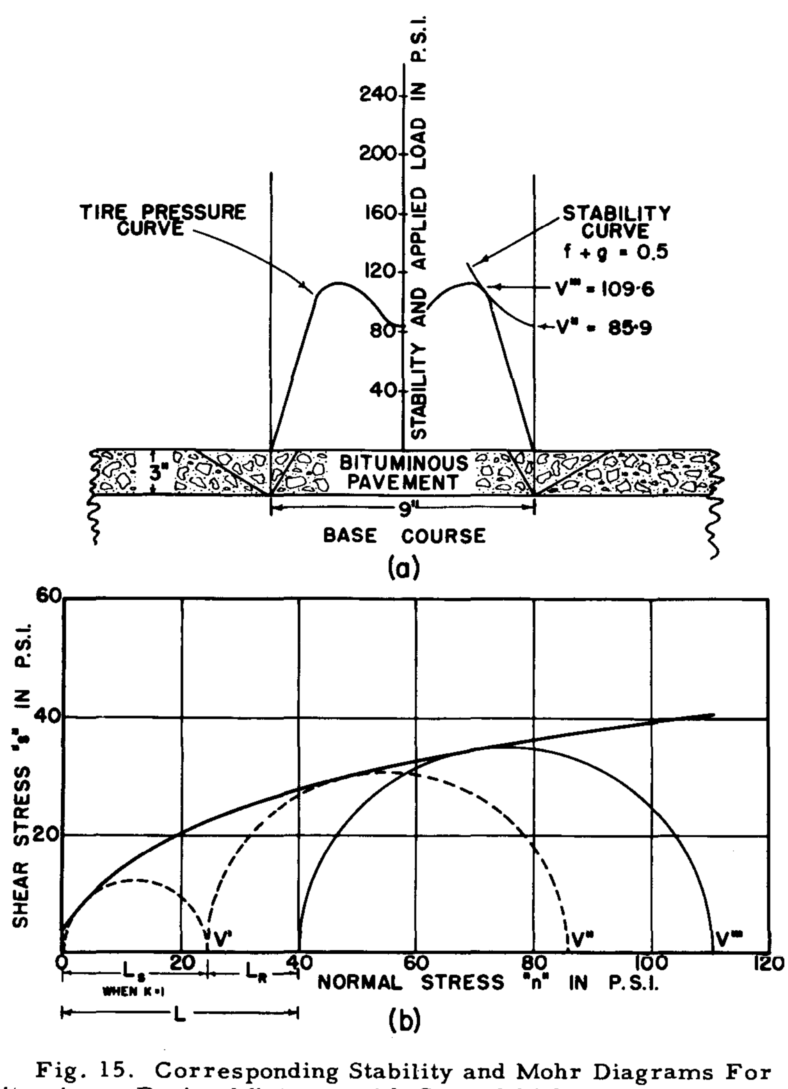


Fig. 15. Corresponding Stability and Mohr Diagrams For Bituminous Paving Mixtures with Curved Mohr Envelopes.

and 8. By plotting these stability values V versus distance inward from the edge of the contact area, and drawing a smooth curve through them, a stability curve like that of Figure 15(a) is obtained.

Step No. 10

By comparing the stability curve with the tire pressure curve, e.g. Figure 15(a), it can be quickly determined whether or not the pavement will be stable under the applied load. If the stability curve cuts through the tire pressure curve at any point, pavement instability can be anticipated since this indicates that the applied load is greater than the pavement stability over part of the contact area. If the stability curve is tangent to or above the tire pressure curve for all points on the contact area, the pavement stability is equal to or greater than the applied load, and a stable pavement could be normally expected.

COMPARISON OF STABILITY VALUES FOR CURVED VERSUS STRAIGHT MOHR ENVELOPES

Figure 16 indicates the degree of error to be expected if an engineer should consider drawing the best straight line through the triaxial data of the principal stress diagram of Figure 16(a) or Figure 12, and treating the data as though it provided a straight rather than a curved Mohr envelope. The method of least squares was employed to locate the best straight line through the data of Figure 16(a). From Figure 16(a) the straight and curved line Mohr envelopes for the corresponding Mohr diagram of Figure 16(b) were obtained.

For both curved and straight Mohr envelopes of Figure 16(b), Mohr circles are compared for which the lateral support L is equal to 1.6 times the unconfined compressive strength. For the straight Mohr envelope, it will be noted that the stability V_s = 142.4 p.s.i., while for the curved Mohr envelope the stability V_c is only 110.2 p.s.i. Consequently, by assuming a straight Mohr envelope for the triaxial data of Figure 16(a), instead of the curved Mohr envelope that they actually represent, an engineer would over-estimate the stability of the bituminous pavement by very nearly 30 per cent.

SUMMARY

(1) A rational method for bituminous pavement design for bituminous mixtures with straight Mohr envelopes is briefly reviewed.

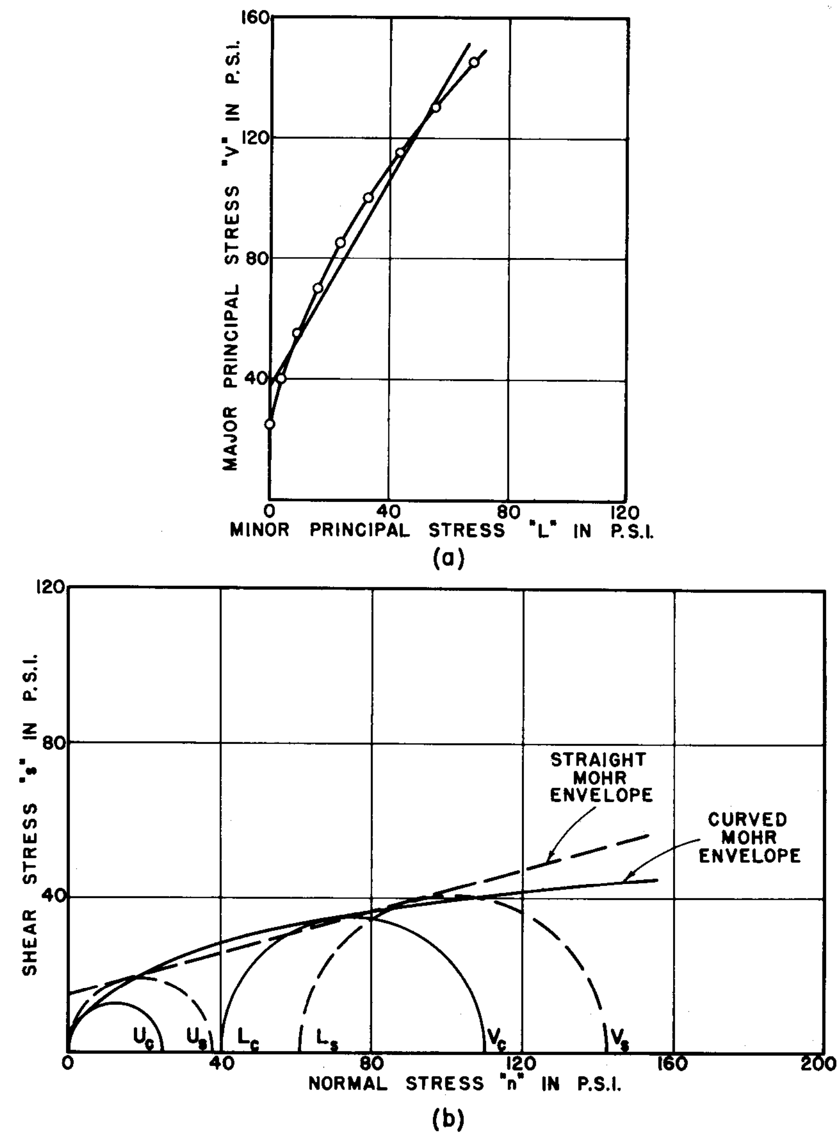


Fig. 16. Comparison of Stability Values Given By the Best Curved Line Through V and L Values Plotted On a Principal Stress Diagram (Resulting in a Curved Mohr Envelope) Versus Those Provided By the Best Straight Line Through the Same Points (Resulting in a Straight Mohr Envelope).

(2) When all other factors are equal, it is shown that the stability of a bituminous pavement depends very materially on the lateral support provided by the portion of the pavement adjacent to the loaded area, and upon the frictional resistances between pavement and tire and between pavement and base.

(3) A rational method for bituminous pavement design for bituminous mixtures with curved Mohr envelopes is described, assuming that the curved Mohr envelope can be represented by an exponential equation.

(4) By means of an example, it is shown that by employing the best straight Mohr envelope for a bituminous mixture for which the triaxial data actually plot as a curved Mohr envelope, the stability of the paving mixture might be overestimated by as much as 30 per cent.

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APPENDIX

Analysis of Mohr Diagrams with Curved Mohr Envelopes

For the Mohr diagrams of Figures 13 and 14, it is assumed that the curved Mohr envelope can be represented by the exponential equation

$$s + d = m \log (n + a) \tag{18}$$

where

s and n are shear and normal stress, respectively, and d, m, and a are constants.

In Figures 13 and 14, the equation for Mohr circle (1) representing the unconfined compressive strength is

$$\left(n - \frac{U}{2}\right)^2 + s^2 = \left(\frac{U}{2}\right)^2$$

which, upon rearranging and simplifying, becomes

$$s^2 = nU - n^2 \tag{19}$$

where

U = unconfined compressive strength, and
n and s = normal and shear stress, respectively, on any plane through the test specimen subjected to the unconfined compressive strength U.

In Figure 13, the equation for Mohr circle (2), for which the lateral support L_2 is equal to the unconfined compressive strength U, is

$$\left[n - \left(\frac{V_2 + U}{2}\right)\right]^2 + s^2 = \left(\frac{V_2 - U}{2}\right)$$

which upon rearranging and simplifying becomes

$$s^2 = n\,(V_2 + U) - n^2 - V_2\,U \tag{20}$$

where

V_2 = the major principal stress when the lateral support L_2 is equal to the unconfined compressive strength U, and
n and s = normal and shear stresses on any plane through the specimen when the principal stresses are V_2 and U.

In Figure 14, the equation for any Mohr circle (3), for which the principal stresses are V_3 and L_3, where L_3 = KU, is

$$\left[n - \left(\frac{V_3 + L_3}{2}\right)\right]^2 + s^2 = \left(\frac{V_3 - L_3}{2}\right)^2$$

which, upon rearranging and simplifying, becomes

$$s^2 = n\ (V_3 + L_3) - n^2 - V_3L_3 \tag{21}$$

where

V_3 and L_3 = major and minor principal stresses, and L_3 = KU, and
n and s = normal and shear stresses on any plane through the specimen when the principal stresses are V_3 and L_3.

In Figures 13 and 14, the slope of the tangent at any point on the curved Mohr envelope is given by the first derivative of equation (18),

$$\frac{ds}{dn} = \frac{0.4343\ m}{n + a} \tag{22}$$

In Figure 13 and 14, the slope of the tangent at any point on the circumference of Mohr circle (1) is given by the first derivation of equation (19)

$$\frac{ds}{dn} = \frac{U - 2n}{2s} \tag{23}$$

In Figure 13, the slope of the tangent at any point on the circumference of Mohr circle (2) is given by the first derivative of equation (20),

$$\frac{ds}{dn} = \frac{V_2 + U - 2n}{2s} \tag{24}$$

In Figure 27, the slope of the tangent at any point on the circumference of Mohr circle (3) is given by the first derivative of equation (21),

$$\frac{ds}{dn} = \frac{V_3 + L_3 - 2n}{2s} \tag{25}$$

Analysis of Mohr Circle (1)
Representing Unconfined Compression Figures 13 and 14

For Mohr circle (1) in Figures 13 and 14, the unconfined compressive strength U is to be evaluated and values can be determined for n_u and s_u, the normal and shear stress coordinates for the point of tangency G between the curved Mohr envelope and Mohr circle (1), and for c_u and $\emptyset_u$ given by the tangent to the Mohr envelope at G.

At the point of tangency G between the curved Mohr envelope and Mohr circle (1), the equation for the curved Mohr envelope is

$$s_u + d = m \log\ (n_u + a) \tag{18a}$$

and the equation for Mohr circle (1) is

$$s_u^2 = n_u U - n_u^2 \tag{19a}$$

Squaring equation (18a), equating it to equation (19a) and rearranging, gives

$$U = \frac{[m \log (n_u + a) - d]^2 + n_u^2}{n_u} \tag{26}$$

At the point of tangency, G, the slopes of the tangents to Mohr circle (1) and to the curved Mohr envelope are equal. Consequently, equating equations (22) and (23), and introducing appropriate subscripts, gives

$$\frac{0.4343\ m}{n_u + a} = \frac{U - 2n_u}{2s_u}$$

which, upon substituting the right-hand side of equation (18a) for s_u, simplifying, and rearranging, becomes,

$$U = \frac{0.8686\ m\ [m \log (n_u + a) - d]}{n_u + a} + 2n_u \tag{27}$$

Equating equations (26) and (27), and simplifying, gives,

$$[m \log (n_u + a) - d]\ [(n_u + a) \{m \log (n_u + a) - d\} - 0.8686\ m\ n_u] - n_u^2 (n_u + a) = 0 \tag{28}$$

In equation (28), d, m, and a are constants from equation (18), which represents the curved Mohr envelope, and values for them are given in Figures 13 and 14. The method for evaluating each of the three constants d, m, and a is illustrated in the example of calculations given later in the appendix. Therefore, since d, m, and a are known, the value for n_u required to satisfy equation (28) can be quickly determined from a graphical plot of equation (28) versus trial values for n_u, Figure 17(b).

By substituting the value for n_u found in this manner in equations (26) and (27), the value for U can be calculated, since the values for the constants d, m, and a have been established.

In addition, since values for n_u, d, m, and a have been determined, values for s_u, ϕ_u, and c_u can be calculated from the following equations:

$$s_u + d = m \log (n_u + a) \tag{18a}$$

$$\phi_u = \tan^{-1} \frac{0.4343\ m}{n_u + a} \tag{29}$$

$$c_u = s_u - n_u \tan \phi_u \tag{30}$$

Because it would add to the detail of the diagrams, the tangent to Mohr circle (1) at its point of tangency with the curved Mohr envelope is not shown in either Figure 13 or 14.

Analysis of Mohr Circle (2), Figure 26

For Mohr circle (2) in Figure 13, for which the lateral pressure L_2 is equal to the unconfined compressive strength U, the major principal stress V_2 is to be evaluated, and values can be determined for n_2 and s_2, the normal and shear stress coordinates for the point of tangency, I, between the curved Mohr envelope and Mohr circle (2), and for c_2 and ϕ_2 given by the tangent to the curved Mohr envelope at I.

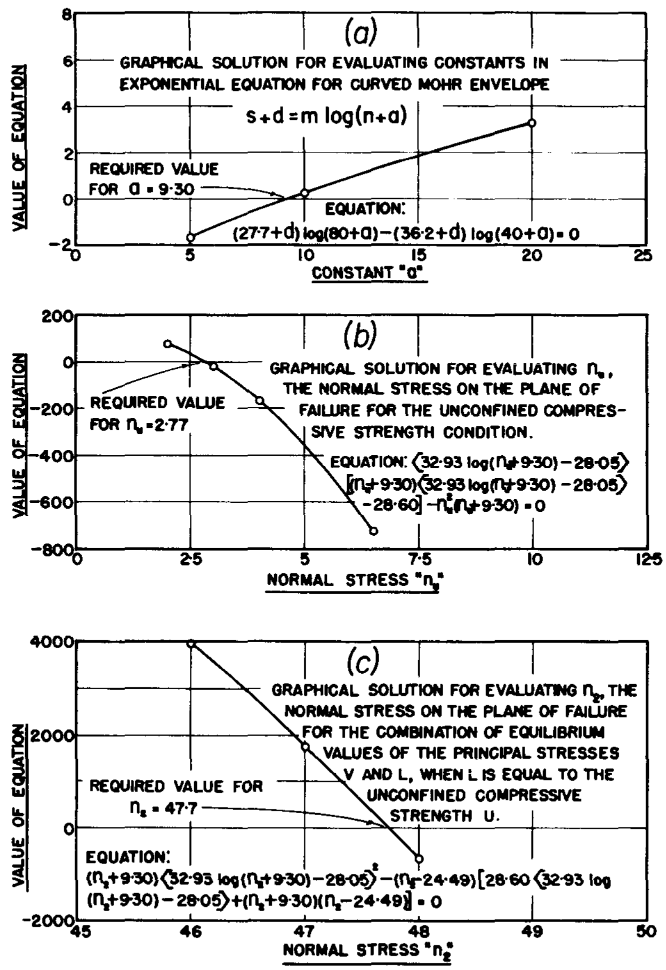

Fig. 17. Illustrating Graphical Methods For Simplifying the Solution of Three Key Equations Associated with Curved Mohr Envelopes.

At the point of tangency, I, the equations for the curved Mohr envelope and for the Mohr circle (2) are, respectively,

$$s_2 + d = m \log (n_2 + a) \tag{18b}$$

and

$$s_2^2 = n_2 (V_2 + U) - n_2^2 - V_2 U \tag{20b}$$

Squaring equation (18b), equating it to equation (20b), and rearranging, gives

$$V_2 = \frac{[m \log (n_2 + a) - d]^2}{n_2 - U} + n_2 \qquad (31)$$

At the point of tangency I, the slopes of the tangents to the curved envelope and Mohr circle (2) are equal. Therefore, equating equations (22) and (24), introducing appropriate subscripts, and rearranging (remembering that $s_2 + d = m \log (n_2 + a)$), gives

$$V_2 = \frac{0.4343\ m\ [2 \{m \log (n_2 + a) - d\}]}{n_2 + a} - U + 2n_2 \qquad (32)$$

Equating equations (31) and (32), and simplifying, gives,

$$(n_2 + a) \{m \log (n_2 + a) - d\}^2 - (n_2 - U) [0.8686\ m \{m \log (n_2 + a) - d\} + (n_2 + a)(n_2 - U)] = 0 \qquad (33)$$

In equation (33), values for the constants d, m, and a from equation (18) for the curved Mohr envelope are given in Figures 13 and 14 (see the example of calculations at the end of this appendix for a method for their evaluation), the value for U has already been determined for Mohr circle (1), and n_2 is, therefore, the only unknown. The value for n_2 required to satisfy equation (33) can be quickly determined from a graphical plot of equation (33) versus trial values for n_2, Figure 17(c).

By substituting the value for n_2 found by this means in equations (31) or (32), the value of V_2 can be calculated, since the values for d, m, a, U, and n have been determined.

In addition, since values for n_2, d, m, and a are known, values for s_2, ϕ_2, and c_2 can be calculated from the following equations:

$$s_2 + d = m \log (n_2 + a) \qquad (18b)$$

$$\phi_2 = \tan^{-1} \frac{0.4343\ m}{n_2 + a} \qquad (34)$$

$$c_2 = s_2 - n_2 \tan \phi_2 \qquad (35)$$

Analysis of Mohr Circle (3), Figure 14

For any Mohr circle (3), Figure 14, for which the lateral pressure L_3 is equal to KU, where KU is the unconfined compressive strength of the paving mixture multiplied by any specified factor K, the major principal stress V_3 is to be evaluated, and values can be determined for n_3 and s_3, the normal and shear stress coordinates for the point of tangency, F, between the curved Mohr envelope and Mohr circle (3), and for c_3 and ϕ_3 given by the tangent to the curved Mohr envelope at F.

At the point of tangency F, the equations for the curved Mohr envelope and for Mohr circle (3), respectively, are

$$s_3 + d = m \log (n_3 + a) \qquad (18c)$$

and

$$s_3^2 = n_3 (V_3 + L_3) - n_3^2 - V_3 L_3 \qquad (19c)$$

Squaring equation (18c), equating it to equation (19c), and rearranging, gives

$$V_3 = \frac{\{m \log (n_3 + a) - d\}^2}{n_3 - L_3} + n_3 \qquad (36)$$

At the point of tangency, F, the slopes of the tangents to the curved envelope and Mohr circle (3) are equal. Therefore, equating equations (22) and (25), introducing the appropriate subscripts, and rearranging (remembering that $s_3 + d = m \log (n_3 + a)$), gives

$$V_3 = \frac{0.4343 \text{ m } [2 \{m \log (n_3 + a) - d\}]}{n_3 + a} - L_3 + 2n_3 \qquad (37)$$

Equating equations (36) and (37), and simplifying, gives

$$(n_3 + a) \{m \log (n_3 + a) - d\}^2$$
$$- (n_3 - L_3) [0.8686 \text{ m } \{m \log (n_3 + a) - d\} + (n_3 + a)(n_3 - L_3)] = 0 \qquad (38)$$

In equation (38), values for the constants d, m, and a from equation (18) for the curved Mohr envelope are given in Figures 13 and 14 (see example of calculations at the end of this appendix for their evaluation), the value of L_3 = KU is known, since the value of K to be used is always specified, and the value of U has already been determined for Mohr circle (1), and n_3 is, therefore, the only unknown. The value for n_3 required to satisfy equation (38) can be quickly determined from a graphical plot of equation (39) versus trial values for n_3, similar to that of Figure 17(c) for n_2.

By substituting the value for n_3 found by this means in equations (36) or (37), the value for V_3 can be found, since the values for d, m, a, L_3, and n_3 have been already determined. It will be remembered that L_3 = KU, for which the value of K is always specified, and the value of U, the unconfined compressive strength, has already been determined for Mohr circle (1).

Since values for d, m, a, and n_3 have been determined, values for s_3, ϕ_3, and c_3 can be calculated from the following equations:

$$s_3 + d = m \log (n_3 + a) \qquad (18c)$$

$$\phi_3 = \tan^{-1} \frac{0.4343 \text{ m}}{n_3 + a} \qquad (39)$$

$$c_3 = s_3 - n_3 \tan \phi_3 \qquad (40)$$

AN EXAMPLE OF CALCULATIONS

To Evaluate the Constants d, m, and a In the Exponential Equation Representing the Curved Mohr Envelope

It is assumed that points representing the V and L values provided by a triaxial test on a bituminous paving mixture with a curved Mohr envelope have been plotted on a principal stress diagram, and that the best smooth curve has been drawn through them, e.g. Figure 12. Using V and L values from well distributed points on this curve, describe a number of Mohr circles and draw a smooth curved envelope tangent to them. Select three points X, Y, and Z on this curved Mohr envelope, Figure 13, of such spacing that point X is slightly to the right of the point of tangency, G, of the curved Mohr envelope with Mohr circle (1) representing the unconfined compressive strength, point Z is somewhat to the right of the point of tangency of the curved envelope with the Mohr circle considered to represent the most critical stability value of the pavement; that is, somewhat to the right of I in Figure 13, and of F in Figure 14; while point Y is approximately half-way between them.

As precisely as they can be read from the curved Mohr envelope of Figure 13, the normal stress and shear stress (n and s) coordinates for points X, Y, and Z are as follows:

	n p.s.i.	s p.s.i.
X	5	10.0
Y	40	27.7
Z	80	36.2

The exponential equation assumed to represent the curved Mohr envelope over the range of stress under consideration is

$$s + d = m \log (n + a) \qquad (18)$$

Equation (18) contains three constants, d, m, and a, and the two variables normal stress n and shear stress s. The three constants d, m, and a can be evaluated by substituting the n and s values for the three points X, Y, and Z in equation (18) to form three equations that can be solved simultaneously. These three equations are:

$$10 + d = m \log (5 + a) \qquad (a)$$

$$27.7 + d = m \log (40 + a) \qquad (b)$$

$$36.2 + d = m \log (80 + a) \qquad (c)$$

Combining equations (a) and (b), and equations (b) and (c), so as to eliminate m, gives,

$$(10 + d) \log (40 + a) = (27.7 + d) \log (5 + a) \qquad (d)$$

and

$$(27.7 + d) \log (80 + a) = (36.2 + d) \log (40 + a) \qquad (e)$$

Substitute trial values for a in equation (d) and calculate corresponding values for d. Substitute the values for a and d so obtained into equation (e), and plot the left-hand side of equation (e) minus the right-hand side against the trial values for a, Figure 17(a). When this difference is zero, the correct value for a has been obtained. Figure 17(a) indicates that the correct value for a = 9.3.

When the correct value for a, 9.3, is substituted in equation (d), it is found that d = 28.05. These values for a and d are also found to satisfy equation (e), which serves as a check.

When the values a = 9.3 and d = 28.05 are substituted in one or more of equations (a), (b), or (c), it is found that the value for the constant m = 32.93.

Therefore, the required values for the constants d, m, and a in equation (18) are

$$d = 28.05$$
$$m = 32.93$$
$$a = 9.3$$

Analysis of Mohr Circle (1), Unconfined Compression, Figures 13 and 14

The value for n_u at the point of tangency, G, between the curved Mohr envelope and Mohr circle (1), Figure 13, where n_u represents the normal stress on the plane of failure for the unconfined compressive strength condition, can be calculated from equation (28),

$$[m \log (n_u + a) - d] [(n_u + a) \{m \log (n_u + a) - d\} - 0.8686\, m\, n_u] - n_u^2 (n_u + a) = 0 \quad (28)$$

Values for the constants d, m, and a have already been determined and the value of n can be found graphically by plotting equation (28) against trial values for n_u until a value for n_u is found that satisfies the equation. Figure 17(b) illustrates this method, and indicates that the required value for n_u = 2.77 p.s.i.

By substituting the values determined for d, m, a, and n_u in equations (26) or (27), the value of U, the unconfined compressive strength, Figures 13 and 14, can be calculated.

$$U = \frac{[m \log (n_u + a) - d]^2 + n_u^2}{n_u} \quad (26)$$

from which

$$U = 24.49 \text{ p.s.i.}$$

From the values for d, m, a, and n_u that have been established, values for s_u, ϕ_u, and c_u can be easily calculated by means of equations (18a), (29), and (30).

$$s_u + d = m \log (n_u + a) \quad (18a)$$

from which

$$s_u = 7.58 \text{ p.s.i.}$$

$$\phi_u = \tan^{-1} \frac{0.4343\, m}{n_u + a} \quad (29)$$

from which

$$\phi_u = 51^\circ 21'$$

$$c_u = s_u - n_u \tan \phi_u \quad (30)$$

from which

$$c_u = 4.11 \text{ p.s.i.}$$

Consequently, for Mohr circle (1) in Figures 13 and 14,

$$U = 24.49 \text{ p.s.i.}$$
$$n_u = 2.77 \text{ p.s.i.}$$
$$s_u = 7.58 \text{ p.s.i.}$$
$$c_u = 4.11 \text{ p.s.i.}$$
$$\phi_u = 51^\circ\ 21'$$

Analysis of Mohr Circle (2), Figure 13

The value for n_2 at the point of tangency I between the curved Mohr envelope and Mohr circle (2), Figure 13, where n_2 represents the normal stress on the plane of failure when the principal stresses are V_2, and $L_2 = U$, can be calculated from equation (33).

$$(n_2 + a) \left\{m \log (n_2 + a) - d\right\}^2 - (n_2 - U) \left[0.8686\ m \left\{m \log (n_2 + a) - d\right\} + (n_2 + a)(n_2 - U)\right] = 0 \qquad (33)$$

Values for the constants d, m, and a, and for the unconfined compressive strength U have already been determined, and the value for n can be found graphically by plotting equation (33) against trial values for n_2 until this equation is satisfied. Figure 17(c) illustrates this method, and indicates that the required value for $n_2 = 47.7$ p.s.i.

By substituting the values determined for d, m, a, U, and n_2 in equations (31) or (32), the value of V_2, the major principal stress, Figure 13, can be calculated.

$$V_2 = \frac{\left[m \log (n_2 + a) - d\right]^2}{n_2 - U} + n_2 \qquad (31)$$

from which

$$V_2 = 85.9 \text{ p.s.i.}$$

From the values for d, m, a, and n_2 that have been established, values for s_2, ϕ_2, and c_2 can be easily calculated by means of equations (18b), (34), and (35).

$$s_2 + d = m \log (n_2 + a) \qquad (18b)$$

from which

$$s_2 = 29.77 \text{ p.s.i.}$$

$$\phi_2 = \tan^{-1} \frac{0.4343\ m}{n_2 + a} \qquad (34)$$

from which

$$\phi_2 = 14^\circ\ 05'$$

$$c_2 = s_2 - n_2 \tan \phi_2 \qquad (35)$$

from which

$$c_2 = 17.8 \text{ p.s.i.}$$

Consequently, for Mohr circle (2) in Figure 26

$$V_2 \equiv 85.9 \text{ p.s.i.}$$
$$U = 24.49 \text{ p.s.i.}$$
$$n_2 = 47.7 \text{ p.s.i.}$$
$$s_2 = 29.77 \text{ p.s.i.}$$
$$c_2 = 17.8 \text{ p.s.i.}$$
$$\phi_2 = 14^\circ\ 5'$$

Analysis of Mohr Circle (3), Figure 14, General Case

The value of n_3 at the point of tangency F between the curved Mohr envelope and Mohr circle (3), Figure 14, where n_3 represents the normal stress on the plane of failure when the principal stresses are V_3, and L_3 = KU, where K = 1.225 as an arbitrarily specified value in this example, and U is the unconfined compressive strength, can be calculated from equation (38),

$$(n_3 + a)\{m \log (n_3 + a) - d\}^2 - (n_3 - L_3)[0.8686\ m\ \{m \log (n_3 + a) - d\} + (n_3 + a)(n_3 - L_3)] = 0 \qquad (38)$$

Values for the constants d, m, and a have already been determined, L_3 = KU, both K and U being known, and the value for n_3 can be found graphically by plotting equation (38) against trial values for n_3, until a value for n_3 is determined that satisfies this equation. The method is illustrated in Figure 17(c), which was employed to evaluate n_2, and a smiliar graph indicates that the required value for n_3 = 55.36 p.s.i.

By substituting the values determined for d, m, a, and n_3 in equations (36) or (37), and remembering that L_3 = KU, the value of V_3, the major principal stress, Figure 14, can be calculated.

$$V_3 = \frac{\{m \log (n_3 + a) - d\}^2}{n_3 - L_3} + n_3 \qquad (36)$$

from which

$$V_3 = 94.68 \text{ p.s.i.}$$

From the values for d, m, a, and n_3 that have been established, values for s_3, ϕ_3, and c_3 can be easily calculated by means of equations (18c), (39), and (40).

$$s_3 + d = m \log (n_3 + a) \qquad (18c)$$

from which

$$s_3 = 31.58 \text{ p.s.i.}$$

$$\phi_3 = \tan^{-1} \frac{0.4343\ m}{n_3 + a} \tag{39}$$

from which

$$\phi_3 = 12^\circ\ 28'$$

$$c_3 = s_3 - n_3 \tan \phi_3 \tag{40}$$

from which

$$c_3 = 19.34 \text{ p.s.i.}$$

Consequently, for Mohr circle (3) in Figure 14,

$$V_3 = 94.68 \text{ p.s.i.}$$
$$L_3 = KU = (1.225)(24.49) = 30 \text{ p.s.i.}$$
$$n_3 = 55.36 \text{ p.s.i.}$$
$$s_3 = 31.58 \text{ p.s.i.}$$
$$c_3 = 19.34 \text{ p.s.i.}$$
$$\phi_3 = 12^\circ\ 28'$$

<u>To Calculate Points On the Stability Curve</u>
<u>between the Edge and the Centre of the Contact Area</u>

Suppose that the stability V is to be calculated at a distance of one-half inch along the transverse axis from the edge toward the centre of the contact area, e.g. Figure 15(a).

The following assumptions are made, and are illustrated in Figure 15.

(a) $f + g = 0.5$
(b) pavement thickness = 3 inches
(c) the curved Mohr envelope for the paving mixture is as illustrated in Figure 15(b)
(d) the tire pressure curve has the shape shown in 15(a), and rises from the edge of the contact area at the rate of 60 p.s.i. per inch of horizontal distance.

The value of L_R at a distance one-half inch inward from the edge of the contact area can be calculated by means of equation (7).

$$L_R = \frac{dV'}{t} (f + g) \tag{7}$$

where

d = distance inward from the edge of the contact area = 0.5 inch
t = thickness of pavement = 3 inches
f + g = 0.5
V' = average tire pressure acting over the ρver-half inch of distance inward from the edge of the contact area = $\frac{30 - 0}{2}$ = 15 p.s.i.

Substituting these values in equation (7) gives,

$$L_R = \frac{(0.5)\ (15)\ (0.5)}{(3)} = 1.25 \text{ p.s.i.}$$

From Figure 14, or from the Appendix, the lateral support provided by the pavement adjacent to the contact area = L_S = 24.49 p.s.i.

The total lateral support L acting at a point one-half inch inward from the edge of the contact area is given by $L = L_S + L_R = 24.49 + 1.25 = 25.74$ p.s.i.

Any value of lateral support L can be expressed as KU, where U = the unconfined compressive strength of the paving mixture = 24.49 p.s.i., Figure 15(b). Therefore, in this case $K = \frac{L}{U} = \frac{25.74}{24.49} = 1.05$.

The pavement stability V at the point one-half inch inward from the edge of the contact area can now be calculated by the procedure outlined in the preceding section of the Appendix, "Analysis of Mohr Circle (3), Figure 14, General Case." Remembering that $L_3 = KU = 25.74$ p.s.i. in this case, it is readily determined that $V_3 = 87.9$ p.s.i. Therefore, the stability of the pavement at 0.5 inch inward from the edge of the contact area, for the conditions outlined, is given by V = 87.9 p.s.i.

Stability values can be similarly calculated for the pavement for points at other distances inward from the edge of the contact area. The smooth curve drawn through the plot of these data constitutes the stability curve of Figure 15(a), which indicates pavement stability at various distances inward from the edge of the loaded area. For the particular conditions on which Figure 15 is based, the stability curve is just tangent to the tire pressure curve at a distance of 1.7 inches inward from the edge of the contact area, Figure 15(a). At this point of tangency, both tire pressure and pavement stability are 109.6 p.s.i.

It should be apparent that for a more stable paving mixture, or for a higher value for f + g, the frictional resistance between pavement and tire and between pavement and base, or for a smaller pavement thickness, etc., the stability curve would lie above its present position; that is, the stability of the pavement would exceed the tire pressure at all points on the contact area.

Discussion

PROFESSOR B. A. VALLERGA: The paper just presented is the fullest treatment of the subject I have heard. It is a mathematical approach to evaluating the amount of load a surface course can sustain from a consideration of the value of the angle of friction, Ø, and the intercept on the Mohr diagram, C, and a number of other factors. I have admired Dr. McLeod's approach very much, and it has merit.

Last year, in a discussion I questioned the curvature of the Mohr envelope, and I still do. I believe that eventually we will find out whether curved Mohr envelopes for asphaltic mixtures do exist. However, Dr. McLeod's approach takes care of both straight and curved envelopes so the matter is not critical.

Dr. McLeod, there is one point I would like to raise. You have referred to this method as a method of designing bituminous mixtures. Earlier today I gave my definition of mix design. Your method, as presented, would not satisfy this definition. It seems to me that by your method one of two things would be done; first, a mix could be designed and then tested to determine the values of Ø and C. Then, by your method one could determine how much load a pavement made of this material should be able to sustain. Secondly, the reverse could be done; knowing the load to which the pavement would be subjected, one could determine the minimum values of Ø and C which the asphaltic pavement should have.

Now I believe this is correct, however, I do not consider that this constitutes the actual design of a bituminous mixture. Could you tell us what you mean when you say this is a method for designing bituminous mixtures?

DR. McLEOD: That is a very good question, Professor Vallerga. I am very glad you brought it up. Possibly rather than "design," the paper should specifically refer to "stability of bituminous pavements." In some of the earlier papers it was emphasized that first of all, a bituminous mixture should be properly designed on every other basis that should be taken into consideration, including, asphalt content, workability, gradation, durability, and so on. When satisfied that it meets design requirements in every other respect, this paper then provides a means for establishing whether or not the paving mixture will have the stability required to carry the anticipated load.

MR. JOHN GRIFFITH: Dr. McLeod, your analysis is predicated on the use of triaxial test data which involve only a single application of load to the specimen to derive the information which you seek. Now, I think we realize that when failures occur in bituminous mats they generally do so under many thousands of repetitions of load. There are some indications from repetitive triaxial testing on soils entirely different results will be observed.

Might it not be well to consider some repetitive type of testing in conjunction with this type of analysis?

DR. McLEOD: That is also a very pertinent point, Mr. Griffith. I believe that a procedure could be established for the triaxial test itself that would provide for repetitive instead of single loadings, thereby satisfying the requirement you have mentioned.

MR. HECTOR M. CALDERON: I have several questions.

One is this: The author has stated that the unconfined compressive test is a measure of the passive resistance of the pavement to displacement. I question that because the unconfined compressive resistance of a bituminous mixture or pavement is measured vertically; it is an active pressure, and if the pavement is a homogeneous material, you could probably use that value as a first approximation. However, I think that most pavements are not isotropic; their particles are oriented more or less horizontally, so you should find resistance in a horizontal direction will be quite different from vertical resistance, as measured by the unconfined compression test.

In the second place, I don't believe that the shear resistance on vertical planes transverse to the loaded lower area are taken into account in Dr. McLeod's theory. That is, he analyzes the problem from a two-dimensional point of view and, actually, in order that the pavement may be pushed out laterally, it has to overcome the resistance of the rest of the pavement in that same direction, that is, on vertical planes transverse to the loaded area.

I would appreciate your answer on those points.

DR. McLEOD: In answer to the suggestion that the lateral pressure might not be equal to the unconfined compressive strength as ordinarily measured, we have recognized that fact and have pointed it out in some of our previous papers. It was not mentioned during the presentation of this paper because of the limitation of time. In these earlier papers we have suggested that the unconfined compressive strength should be multiplied by a factor K to take into account certain items that we realize we have left out. At the present time, precise measured values for K are not available. However we feel that if K were assumed equal to Unity, it would probably be on the conservative side insofar as the ultimate strength of the pavement is concerned.

MR. CALDERON: Would that value of K take into account the fact that the shearing resistance might be different on vertical and horizontal planes of the pavement.

DR. McLEOD: We have suggested it might do so, but that to get the answer you are after it might be necessary to use the type of triaxial test Professor Haefeli uses in Switzerland, where the lateral pressure becomes the principal stress, and the vertical load is the minor stress.

The comments made by Mr. Calderon are greatly appreciated because they touch on important aspects of any rational approach to the design of bituminous pavements.